lest it should be struck by the stem that suspends the gun, which would somewhat check the recoil. After the gun returns from the recoil, stop the vibrations gently, without touching the index, and look what division on the arch the index points to, or stands at, which number will show the strength of the charge. For more satisfaction, the same fired charge may be repeated two or three times, and the medium of all the numbers taken for the strength of the charge.

TRACT XXXVI.

RESISTANCE OF THE AIR DETERMINED BY THE WHIRLING-MACHINE.

ART. 1. THE method of determining the resistance of the air, as detailed in Tract 34, by firing cannon balls into the ballistic pendulum, it was found by experience could only be practised with the higher velocities; since, with those that are below 300 in a second, the balls could hardly ever be made to lodge in the block, but rebounded back, and thus defeated the success of the experiment. To get the resistances then to the smaller velocities, I had recourse to another method, viz, to that of Mr. Robins's whirling-machine, one of which I was fortunate enough to meet with ready made, being probably the identical one with which he made his experiments, or at least it was made by the same workman, under the directions of Mr. Ellicot.

2. With this machine, Mr. Robins made many experiments before the members of the Royal Society, with the view chiefly of confirming these two propositions: "1st.

That the resistance of the air to a 12lb iron ball, moving with a velocity of 25 feet in a second, is not less than half an ounce avoirdupois. 2dly. That the resistance of the air, within certain limits, is nearly in the duplicate proportion of the velocity of the resisted body." This machine is described in the first volume of his Tracts, page 202, &c; whence I shall here borrow the description, nearly in his own words.

3. The two propositions above-mentioned, Mr. R. there says, " are in themselves neither unknown nor doubtful; but yet, as they are the basis of some other assertions, which have been hitherto constantly contested and denied, I thought it requisite to evince their veracity by more unquestioned and simpler methods, than have been hitherto practised: for this purpose, I therefore caused a machine to be made of the form represented in plate 4, which was most excellently executed under the direction of Mr. Ellicot, and was completely fitted for the use intended, by many contrivances, some of them not shown in the drawing, nor necessary to be particularized in this place. For, it is sufficient, for the purpose of the following experiments, to describe its general fabric, by observing that BCDE is a brass barrel moveable on its axis, and so adjusted by means of friction wheels, which are not represented in the drawing, as to have no friction worth attending to. The frame, in which this barrel is fixed, is so placed, that its axis may be perpendicular to the horizon. The axis itself is continued above the upper plate of the frame, and has fastened on it a light hollow cone AFG: from the lower part of this cone there is extended a long arm of wood GH, which is very thin, and cut feather-edged, and at its extremity there is a contrivance for fixing on the body whose resistance is to be investigated, as here the globe P; and, to prevent the arm GH from swaying out of its horizontal position, by the weight of the annexed body P, there is a brace AH of fine wire fastened to the top of the cone, which supports the end of the arm.

"Round the barrel BCDE there is wound a fine silk line, the turns of which appear in the figure, and after this line has taken a sufficient number of turns, it is conducted horizontally to the pulley L, over which it is passed, and then a proper weight M is hung to its extremity. If this weight be left at liberty, it is obvious, that it will descend by its own gravity, and will by its descent turn round the barrel BCDE, together with the arm GH and the body P fastened to it. And while the resistance on the arm GH and the body P is less than the weight M, that weight will accelerate its motion, and thereby the motion of GH and P will increase, and consequently their resistance will increase, till at last this resistance and the weight M become nearly equal to each other. The motion with which M descends, and with which the body P revolves, will not sensibly differ from an equable one. Whence it is not difficult to conceive, that by proper observations made with this machine, the resistance of the body P may be determined.

4. "The most natural method of proceeding in this investigation, is as follows. Let the machine have first acquired its equable motion (which, as will hereafter appear, will be usually attained in 5 or 6 turns from the beginning) and then let it be observed, by counting a number of turns, what time is taken up by one revolution of the body P; then taking off the body P and the weight M, let it be examined, what smaller weight will make the arm GH revolve in the same time, as when P was fixed to it; this smaller weight being taken from M, the remainder is obviously equal in effort to the resistance of the revolving body P; and this remainder being reduced in the ratio of the length of the arm to the semi-diameter of the barrel, will then become equal to the absolute quantity of the resistance. And as the time of one revolution is known, and consequently the velocity of the revolving body; there is thereby discovered, the absolute quantity of the resistance to the given body P, moving with a given degree of celerity.

5. "And note, that to avoid all exceptions, I have gene-

rally chose, when the body P was removed, to fix in its stead a thin piece of lead of the same weight, placed horizontally; so that the weight, which was to turn round the arm GH without the body P, did also carry round this piece of lead. This I did, lest it should be objected that the body P retarded the weight M by its quantity of matter, as well as by its resistance. But mathematicians will easily allow, that there is no necessity for this precaution.

6. " The measures of the parts of this machine, were as follow:

	Inches.
" The diameter of the barrel BCDE, and of the silk string wound round it, was - - - - - - - - - -	2·06
" The length of the arm GH, measured from the axis to the surface of the globe P, was - - - - -	49·5
" The body P, the globe made use of, was of pasteboard, its surface very neatly coated with marble paper, and was not much distant from the size of a 12lb shot, being in diameter - - - - - - - - - - -	4·5
" So that the radius of the circle described by the centre of the globe, was - - - - - - - - - - -	51·75

7. " When this globe was fixed at the end of the arm, and a weight of half a pound was hung at the end of the string at M; it was examined, how soon the motion of the descending weight M, and of the revolving globe P, would become equable as to sense. And with this view 3 revolutions being suffered to elapse, it was found, that

The next 10 were performed in -	$27''\frac{3}{4}$
20 in less than - - -	55
30 in - - - - - - -	$82\frac{1}{2}$
So that the first 10 were performed in	$27\frac{3}{4}$
2nd 10 in - - - - -	$27\frac{1}{4}$
3rd 10 in - - - - -	$27\frac{1}{2}$

" These experiments sufficiently evince, that even with half a pound, the smallest weight hereafter used, the motion of the machine was sufficiently equable after the first 3 revolutions.

8. " And now, to prove the two forementioned propositions, the following experiments were made; the times mark ed down being observed by several stop-watches, which rarely differed half a second from each other.

" The forementioned globe being fixed at the end of the arm, there was hung on in the situation M a weight of $3\frac{1}{4}$lb, and 10 revolutions being suffered to elapse, the succeeding 20 were performed in $21''\frac{1}{2}$.

" Then the globe being taken off, and a thin plate of lead, equal to it in weight, placed in its room; it was found, that instead of $3\frac{1}{4}$lb, a weight of 1lb would make it revolve in less time than it did before, it performing 20 revolutions, after 10 were elapsed, in the space of 19 seconds.

" Hence then it follows, that from the $3\frac{1}{4}$lb first hung on, there is less than 1lb to be deducted for the resistance on the arm, and consequently the resistance on the globe itself is not less than the effort of $2\frac{1}{4}$lb in the situation M; and it appearing from the former measures, that the radius of the barrel is nearly $\frac{1}{50}$ part of the radius of the circle described by the centre of the globe; it follows, that the absolute resistance of the globe, when it revolves 20 times in $21''\frac{1}{2}$ (which, if computed by the measures given above, comes out a velocity of about $25\frac{1}{4}$ feet in a second) it follows, I say, that the resistance of the globe in this case is not less than the $\frac{1}{50}$ part of $2\frac{1}{4}$lb, or than the $\frac{1}{50}$ part of 36 ounces: and this being considerably more than half an ounce, and the globe being nearly the size of a 12-pounder; it irrefragably confirms our first proposition, ' That the resistance of the air to a 12lb iron bullet, moving with a velocity of 25 feet in a second, is not less than half an ounce avoirdupois.'

9. " The next experiments were made with a view of examining the 2nd proposition. And, for this purpose, there were successively hung on, in the situation M, weights in the proportion of the numbers, 1, 4, 9, 16; and letting 10 revolutions first elapse, the observations on those that follow, were as follow.

"With 1lb, the globe went 20 turns in $54''\frac{1}{2}$
that is, it went 10 turns in $27\frac{1}{4}$
With 2lb, it went . . 20 turns in $27\frac{1}{2}$
With $4\frac{1}{2}$lb, it went . . 30 turns in $27\frac{1}{2}$
With 8lb, it went . . 40 turns in $27\frac{1}{2}$

"So that it appears, that to the resistances of the proportions of the numbers 1, 4, 9, 16, there correspond velocities of the resisted body, in the proportion of the numbers 1, 2, 3, 4; which proves with great nicety, within the limits of these experiments, the 2nd proposition, 'That the resistance of the air, is nearly in the duplicate proportion of the velocity of the resisted body. That is, it is 4 times as much, when the resisted body moves with twice the velocity; 9 times as much, when it moves with 3 times the velocity, and so on'."

10. Having thus given Mr. Robins's description of the whirling-machine, and the experiments that he performed with it, which will serve sufficiently to bring us acquainted with the nature of it; proceed we now to take account of my own numerous and varied experiments made with the same, in the years 1786 and 1787.——The machine having been cleaned, and put in proper order, so as to cause it to act very perfectly; I procured to be made three hemispheres of pasteboard, of different sizes; both to try the resistance to bodies of different magnitudes, and the difference of resistance between the plane circle and the convex side of the hemisphere. The weight and dimensions of these three hemispheres were as below.

N°	Diam. inches	Area of circle, in		Weight.	
		sqr. inc.	sqr. feet	oz	dr
1	4·75	17·7	$\frac{1}{8}$	2	5
2	5·1	21·2	$\frac{5}{34}$	2	11
3	6·375	32·0	$\frac{2}{9}$	4	3

Having made also three small and thin bits of lead, shaped feather-edged, to fix on the arm in a horizontal position, when each hemisphere was taken off, to find the resistance to the arm, and to obviate any objections, for the same reason as Mr. Robins had done before; the arm was fixed on, with the brass wire, strained from the top of the axis to the end of the arm, to sustain it in the horizontal position.

May 1, 1786.

11. Began to take the necessary dimensions of the parts of the machine. Measured the diameter of the axis two different ways, and each way several times over, taking a mean of them all. One way was by means of a sliding shot-gage, which measured diameters very nicely to hundredths of an inch, by taking the diameter between two parallel arms, made to slide along a stem, divided into inches and 10ths, and these into 10ths again, or the whole into 100ths of inches, by means of a nonius. In this way I took, several times over, and always the same, the diameter of the bare cylinder, without the silk thread wrapped, and then again the diameter including the silk thread on both sides, which is 2 diameters of the thread more than the bare cylinder; viz,

Diameter of the cylinder alone	2·06 inc.
Ditto and double diam. of the thread . .	2·11
Theref. diam. of cyl. and 1 diam. of thread	2·085
And radius to middle of the thread . .	1·042

The other way was, by making the machine turn round exactly a certain number of times, and measuring the space descended by a weight at the end of the thread, as it unwound from the cylinder; then the space so descended being divided by the number of turns, gave the length of one round or circumference of the cylinder and thread; and this again divided by 3·1416, gave the diameter of the cylinder and thread included. This operation being often repeated, and always nearly the same, the medium of all gave for the diam. of cyl. and thread 2·092, therefore the radius of the same is 1·046, which is only ·004 part of an inch more than that by the former method; and the medium of the two is 1·044.

May 2, 1786.

Barometer 30·05; Thermometer 51.

12. Measured from the axis of the cylinder to the centre of each hemisphere, when fitted on the arm, and found them as below, on the line of radii; thence are found the following lines of circumferences, &c.

	N° 1.	N° 2.	N° 3.
	inc.	inc.	inc.
Radii . .	52·42	52·74	53·34
Diams. . .	104·84	105·48	106·68
Circum. .	329·36	331·38	335·14
Ditto feet .	27·47	27·62	27·93
Ratio of radii	50·2	50·5	51·1

The last line being the ratio of each of those radii to that of the barrel and thread 1·044, or the former divided by the latter.

The hemisphere n° 3 was then fitted on, and actuated by a small weight of only $1\frac{3}{4}$oz, or 1oz 12dr, that is $1\frac{3}{4}$oz suspended at the end M of the thread; opposing first the round side to the air, and then the flat side; when the number of rounds, and corresponding times, were observed as follows, both now, and in all the other experiments, by stop-watches, and a peculiar clock made for the purpose.

Hemisphere n° 3, with Weight at M = 28 dr = 1 oz 12 dr.

Plane Side Foremost.		
Turns	Time	Diffs.
1	0′ 27″	
2	· 47	20
3	1 6	19
4	· 24	18
5	· 42	18
6	2 0	18
7	· 18	18
8	· 36	18
9	· 54	18
10	3 12½	18½
11	· 30½	18
12	· 48	17½
13	4 5	17
14	· 22½	17½
1	18	means
$\frac{1}{18}$	1	means
or 1·552 ft.	1	means

Convex Side Foremost.		
Turns	Time	Diffs.
3	0′ 59″	
6	1 37	38
9	2 12	35
12	2 47	35
15	3 22	35
3	35	means
1	11⅔	means
$\frac{3}{35}$	1	means
2·349 ft.	1	means

Note, that the resistance of the arm is still wanted to be deducted from these resistances, and then to be reduced from the circumference of the barrel to the centre of the hemisphere.

In these tables, the first column contains the number of turns made by the arm; the 2nd the corresponding times elapsed from the beginning; and the 3rd shows the differences of those times, answering to the differences of the turns in the first column.

May. 3, 1786.

Barometer 29·63; Thermometer 51.

13. Fitted on the arm the thin bit of lead only n° 3, of the same weight with the largest hemisphere; and after some time spent in levelling the instrument, (which was always done) suspended different small weights at the end of the thread, to discover which weight would give to the arm the same velocities as it had on the day before, when the hemisphere was on. It was found that it required about 10 drams

just to overcome the friction and vis inertia, and give a small motion to the machine. With 12 drams the average of the turns was from $11\frac{1}{2}$ to 12″ each, being nearly the same velocity as the convex side of the hemisphere had with the weight of 12 drams. But with the weight of 1 oz 8 dr = 24 dr, the average turn of the arm this day was made in about 6″. Here then each turn is made in about 12″ with 12 dr weight, and in about 6″ with 24 dr; that is, in this case, the double weight carries it about in nearly half the time. And hence it may be inferred, that a weight of about 8 dr would carry it round once in 18″, being the time of revolving the day before, with the plane of the circle foremost.

Hence then 16 dr or 1 oz, which is the difference between 28 and 12, is the reistance of the air, measured only at the circumference of the barrel, against the convex side of the large hemisphere, moving with a velocity of 2·394 feet in a second; and 20 dr, or $1\frac{1}{4}$ oz, being the difference between 28 and 8 dr, is the relative resistance of the air against the plane side, moving with a velocity of 1·552 feet per second. And each of these divided by 51·1, will give the absolute resistance acting at the centre of the body.

May 4, 1786.

Barometer 29·47; Thermometer 51.

14. Tried several weights actuating the hemisphere, with the plane side foremost, to give it the same velocity as on the 2nd of May, with the convexity foremost, viz, the medium of 1 turn in $11\frac{2}{3}$ seconds. In each trial, the motion became uniform after the first two or three turns; after which, it was permitted to go 12 times round, and the time noted down at every round. The differences of these times commonly agreed with the mean time, to half a second or less. But the mean time itself was very accurately obtained, by dividing the whole observed time of the equable rounds, by 12, the number of them. Which operations, with the times and weights employed this day, are as follows.

Actuating weights		Time of 12 turns	Time of 1 turn
oz	dr		
2	8	158″	13″$\frac{1}{6}$
2	9$\frac{1}{2}$	146$\frac{1}{2}$	12 $\frac{1}{5}$
2	10$\frac{1}{2}$	142	11 $\frac{5}{6}$
2	11	141$\frac{1}{2}$	11 $\frac{3}{4}$
2	12	140	11 $\frac{2}{3}$

Hence then, with the velocity of 1 turn in 11$\frac{2}{3}$ seconds, the arm and plane side of the body are resisted with the force or weight of 2 oz 12 dr; but, on the 2d of May, the resistance to the arm and round side of the body was 1 oz 12 dr; and, on the 3d of May, the resistance to the arm alone was 12 dr; all with the same velocity nearly. Subtracting now 12 dr, the resistance on the arm, from each of the other two, there remain 2 oz for the resistance to the flat side, and 1 oz nearly for that on the convex side; the latter being nearly the half only of the former, and therefore, in this case, nearly agreeing with the theory.

Dividing now this 2 oz by 51·1, the ratio of the radii of the barrel and centre of the body, there results $\frac{20}{511} = \cdot 039$ oz, for the absolute resistance of the flat surface of a circle, moved perpendicularly through the air, with a velocity of 2·394 feet in a second, the area of the circle being 32 square inches, or $\frac{2}{9}$ of a square foot.

May 8, 1786.

Barometer 29·66; Thermometer 56.

15. Tried several different weights, to bring the large hemisphere flat side, also the arm without it, to revolve each time in 10 seconds. Then again, the same body, both sides foremost, and the arm alone, all in 8 seconds, each turn: The particulars are as follows.

For 10 seconds.					For 8 seconds.			
Weights	Plane side		Arm only		Weights	Plane side	Round side	Arm alone
	12 turns	1 turn	12 tns	1 trn				
oz dr								
3 0	140″	$10\frac{5}{6}$	″	″	4 10	8″	″	″
3 2	126	$10\frac{1}{2}$			6 0	7		
3 4	123	$10\frac{1}{4}$			8 0	6		
3 $5\frac{1}{2}$	120	10			2 2		10	
1 0			108	9	2 14		8	
0 14			132	11	1 3			8
0 15			120	10	1 8			6

In the latter of these two tables, the first column shows the actuating weights, and the other three columns the number of seconds, at a medium, in which 1 revolution was made, with each of the two sides of the hemisphere, and the lead alone.

Now, by deducting each weight without the hemisphere, from the corresponding weight with it, there remain the weights for its two sides, as below.

For 10 sec.		For 8 sec.	
3 $5\frac{1}{2}$	2 2	4 10	2 14
0 15	0 15	1 3	1 3
2 $6\frac{1}{2}$	1 3	3 7	1 11

Where the former is nearly double of the latter, in each case, agreeably to theory.

May 9, 1786.

Barometer 29·56; Thermometer 55.

16. Used the large hemisphere with the round side foremost. The weights and times of each revolution as below.

Weight	Time	Weight	Time	Weight	Time	Weight	Time
oz dr	sec.	oz dr	sec.	oz dr	sec.	oz dr	sec.
4 10	$5\frac{5}{6}$	6 0	$4\frac{11}{12}$	7 12	$4\frac{1}{6}$	12 7	$3\frac{5}{24}$
4 9	$5\frac{3}{4}$	6 5	$4\frac{3}{4}$	8 1	$4\frac{1}{12}$	12 10	$3\frac{1}{4}$
4 11	$5\frac{5}{6}$	7 0	$4\frac{1}{2}$	11 5	$3\frac{3}{8}$	13 0	$3\frac{1}{6}$
5 12	5	7 8	$4\frac{1}{4}$	12 0	$3\frac{7}{24}$		

May 10, 1786.

Barometer 29·60; Thermometer ——

17. The large hemisphere with the round side again. The motion was always uniform after the first or second turn. The number of turns always 15, and the medium of the last 12 uniform turns is set down here below opposite the corresponding weights, the same as was done every day, and the titles of the columns being the same as in the foregoing table.

Weight		Time	Weight		Time	Weight		Time	Weight		Time
oz	dr	sec	oz	dr	sec	oz	dr	sec	oz	dr	sec
8	5	$4\frac{1}{12}$	9	11	$3\frac{9}{12}$	13	12	$3\frac{1}{12}$	15	8	$2\frac{10}{12}$
8	8	$4\frac{1}{24}$	10	1	$3\frac{8}{12}$	14	0	3	16	0	$2\frac{9}{12}$
8	12	4	10	8	$3\frac{7}{12}$	14	5	3	16	8	$2\frac{9}{12}$
9	0	$3\frac{11}{12}$	11	0	$3\frac{6}{12}$	14	10	$2\frac{11}{12}$	17	0	$2\frac{9}{12}$
9	6	$3\frac{10}{12}$	13	8	$3\frac{1}{12}$	15	0	$2\frac{11}{12}$	17	8	$2\frac{8}{12}$

May 11, 1786.

Barometer 29·64; Thermometer 60.

18. The large hemisphere again with the round side: every thing else as on the last day.

Wt.	Time	Wt.	Time	Wt.	Time	Wt.	Time
oz	sec	oz	sec	oz	sec	oz	sec
18	2·67	28	2·12	38	1·79	58	1·46
19	2·58	29	2·10	40	1·75	60	1·42
20	2·50	30	2·	42	1·75	64	1·33
21	2·42	31	2·	44	1·67	68	1·33
22	2·42	32	2·	46	1·58	72	1·29
23	2·33	33	1·92	48	1·54	76	1·25
24	2·25	34	1·92	50	1·54	80	1·21
25	2·25	35	1·83	52	1·50	84	1·17
26	2·25	36	1·83	54	1·50	88	1·17
27	2·17	37	1·83	56	1·50	92	1·13

With the next weight, viz, 96 ounces, the silk thread broke twice: so that 96 oz, or 6lb, was just the limit of its strength. It was not woven, but only a twisted cord, framed like a rope, of 3 strands; and was of $\frac{1}{50}$ of an inch in diameter.

May 15, 1786.

Barometer 30·27 ; Therometer 62.

19. Procured a stronger silken cord, and applied it to the following experiments. By measuring the diameters, &c, as before, on the 1st of May, the dimensions came out as below.

Diameter of cylinder alone	. .	2·06
Ditto and 1 diameter of thread	.	2·086
Ditto by descending weight	. .	2·088
Medium of both ways, is	. . .	2·087
Theref. the radius is as before	.	1·044 nearly.

Then continued the last experiments, by increasing the actuating weight, from where they left off the last day, with the convex side, as below.

Weight	Time	Weight	Time	Weight	Tim[e]	Weight	Time
oz	sec.	oz	sec.	oz	sec.	oz	sec.
96	1·12	108	1·04	116	1	124	1
100	1·08	112	1·04	120	1	128	0·96
104	1·08						

So that 120 oz may be presumed to give 1 turn in 1 sec.

The same continued still with the round side.

oz	dr	sec.	oz	dr	sec.	oz	dr	sec.	oz	dr	sec.
4	10	5·8	3	7	6·9	2	10	8·2	2	0	10·2
	8	5·9		6	7·0		9	4	1	15	6
	6	6·0		4	1		8	6	1	14½	8
	4	1		2	3		7	8	1	14	11·1
	2	2		0	5		6	9	1	13½	3
	0	3	2	15	6		5	9·1	1	13	6
3	14	4		14	7		4	3	1	12½	9
	12	6		13	8		3	5	1	12	12·2
	10	7		12	9		2	7	1	11½	6
	8	8		11	8·0		1	9	1	11	13·0

Note, That the machine made always 15 turns, and the time of each turn was carefully noted. Then the time of the 5th turn being taken from that of the 15th, left the time of 10 turns; and this by pointing off the first figure for a decimal, gave the time of 1 turn registered in the table as above.

20. *May* 16, 1786.

Barometer 30·14 ; Thermometer 61.

Began with the flat side of the large hemisphere, in order to go regularly through all the intervals of rotation, from 12 sec. to 1 sec.

Weight	Time	Weight	Time	Weight	Time	Weight	Time
oz dr	sec.	oz dr	sec.	oz dr	sec.	oz dr	sec.
2 10	13·1	2 12	12·1	3 0	11·4	3 6	10·5
10½	12·7	13	11·9	1	11·2	8	10·3
11	12·5	14	11·8	2	10·9	10	10·0
11½	12·3	15	11·6	4	10·7	12	9·6

21. *May* 17, 1786.

Barometer 30·00 ; Thermometer 60.

3 14	9·3	4 12	8·3	5 10	7·5	6 8	6·7
4 0	9·1	14	8·2	12	7·4	12	6·6
2	9·0	5 0	8·1	14	7·2	7 0	6·5
4	8·9	2	7·9	6 0	7·1	4	6·3
6	8·7	4	7·8	2	7·0	8	6·1
8	8·6	6	7·7	4	6·9	12	6·0
10	8·5	8	7·6	6	6·8	8 0	6·0

22. *May* 18, 1786.

Barometer 30·13 ; Thermometer 61.

8 4	6·0	12 0	4·8	21	3·6	44	2·45
8	5·9	13	4·6	22	3·5	48	2·35
12	5·8	14	4·5	24	3·3	52	2·25
9 0	5·7	15	4·3	26	3·2	56	2·15
8	5·5	16	4·1	28	3·1	60	2·05
10 0	5·4	17	4·0	30	3·0	64	2·0
8	5·2	18	3·9	32	2·9	68	1·9
11 0	5·1	19	3·8	36	2·7		
8	5·0	20	3·7	40	2·55		

23. *May* 22, 1786.

Barometer ; Thermometer .

72	1·9	80	1·8	100	1·65	124	1·45
74	1·85	84	1·75	108	1·6	132	1·4
76	1·85	92	1·7	116	1·5	140	1·4

After the last number, the thread broke with the 140 oz in winding up, which interrupted the experiments.

24. *May* 23, 1786.

Barometer 30·20; Thermometer 59.

Having completed the experiments with both sides of the largest hemisphere, began now the similar ones with the lead only of the same weight, the results of which are below.

Weight		Time	Weight		Time	Weight		Time	Weight		Time
oz	dr	sec.	oz	dr	sec.	oz	dr	sec.	oz	dr	sec.
1	0	19	1	6	7½	1	8	6½	1	10	5½
1	2	11	1	7	7	1	9	6	1	12	5
1	4	9									

25. *May* 24, 1786.

Barometer 30·20; Thermometer 61.

1	14	5·0	4	0	2·9	8	8	1·9	22	0	1·1
2	0	4·7	4	8	2·75	9		1·8	24		1·1
2	4	4·4	5	0	2·6	10		1·75	26		1·1
2	8	4·0	5	8	2·45	11		1·65	28		1·0
2	12	3·7	6	0	2·3	12		1·55	30		1·0
3	0	3·5	6	8	2·2	14		1·45	32		1·0
3	4	3·3	7	0	2·1	16		1·35	34		0·9
3	8	3·1	7	8	2·0	18		1·25			
3	12	3·0	8	0	1·95	20		1·2			

26. Having now obtained a long series of actuating weights and velocities, with both sides of the hemisphere foremost, and with the lead alone; a set of the corresponding ones are extracted from the whole, and arranged together, in the first of the following tables; where the first column contains the several rates of velocity, from 3 to 20 feet in a second, differing always by 1; the next 3 columns containing the correspondent weights acting on the axis, of 1·044 radius, for both sides of the body and the lead. But the differences of these not being p.rfectly regular, though very nearly so; correcting therefore those differences here and there, by some very small fractions, and by them the few

irregular weights, the more accurate series of weights are then arranged in the next 3 columns, which are to be considered as the true numbers. Then, subtracting the actuating weights with the lead, from each of the corresponding weights with both sides of the hemisphere, the differences are set in the next two columns, which contain the true weights acting on the axis. And finally, dividing each weight for the flat side, by the corresponding weight for the round side, the quotients are placed in the last column of the table, showing the ratio between the resistances on the two sides.

Velocity.	Irregular weights.			Regular weights.			Diffs. or true wts.		Ratios
	flat	round	lead	flat	round	lead	flat	round	
feet	oz	oz	oz	oz	oz	oz	oz	oz	
3	3·8	2·2	1·2	3·8	2·3	1·2	2·6	1·1	2·37
4	6·1	3·4	1·4	6·2	3·4	1·4	4·8	2·0	2·40
5	9·3	4·9	1·7	9·2	4·9	1·7	7·5	3·2	2·35
6	12·7	6·6	2·0	12·8	6·7	2·0	10·8	4·7	2·30
7	17	8·7	2·5	17·0	8·7	2·4	14·6	6·3	2·31
8	22	11·0	3·0	21·9	11·0	2·9	19·0	8·1	2·35
9	28	13·3	3·5	27·6	13·5	3·5	24·1	10·0	2·41
10	34	15·8	4·2	34·0	16·2	4·2	29·8	12·0	2·48
11	40	19	5·2	41·0	19·2	5·0	36·0	14·2	2·53
12	48	23	6·0	48·7	22·6	5·9	42·8	16·7	2·56
13	56	27	6·8	57·1	26·4	6·8	50·3	19·6	2·57
14	64	31	7·8	66·2	30·6	7·8	58·4	22·8	2·56
15	73	35	8·6	76·0	35·1	8·9	67·1	26·2	2·56
16	84	40	10	86·6	40·0	10·0	76·6	30·0	2·55
17	101	45	11	98·2	45·3	11·2	87·0	34·1	2·55
18	112	50	12	111	51·0	12·4	98·6	38·6	2·55
19	121	57	14	125	57·2	13·7	111·3	43·5	2·56
20	140	64	15	140	64·0	15·0	125·0	49·0	2·55

But note, that all these weights must be divided by 51·1, to reduce them to the centre of the ball.

27. By the last column it appears, that the resistance to the flat side, is to the resistance on the round side, on an average, as 2·48 to 1, or nearly as $2\frac{4}{9}$ to 1; instead of 2 to 1, as expected by the theory.—From the two columns for

both the flat and round sides, it appears, that their resistances increase in a ratio a little higher than that of the square of the velocity: but how much higher, will be determined hereafter, as well as several other inferences.

28. *June* 26, 1786.

Barom. 29·96 ; Thermom. ; very warm.

Having previously got prepared, for the remaining experiments, a flat disc, or very short cylinder, also a complete globe, and a cone, all of pasteboard, and of the same diameter of the preceding largest hemisphere; I proceeded to try, whether a different figure hindmost, as well as foremost, would make any difference, and what, in the results: thus, the hemisphere, the cylinder, and the cone, each with the circle foremost, have three different figures hindmost; and vice versa: also the sphere and hemisphere, with the round side foremost, have different figures hindmost; all of which circumstances will make a great variety of cases. Had the apparatus all well adjusted; the instrument nicely levelled and fixed; and had the upper pulley, over which the cord of the weight passed, fixed so high, as to have 30 or 40 turns completed, before the actuating weight could reach the floor; which would give the medium more exactly of a great number of rounds. The following numbers are the mediums, each of 30 rounds, found by subtracting the time of the 3rd round from that of the 33rd, and dividing the remainder by 30.—The cylinder was used this day, and consequently a flat side both before and behind. Its weight was 5 oz 1 dr.

Weight		Time	Weight		Time	Weight		Time	Weight		Time
oz	dr	sec.	oz	dr	sec.	oz	dr	sec.	oz	dr	sec.
2	0	15·10	3	4	10·57	3	12	9·50	4	12	8·23
2	2	14·40	3	6	10·32	4	0	9·07	5	0	7·93
3	0	11·13	3	8	9·90	4	4	8·83	5	4	7·72
3	2	10·85	3	10	9·73	4	8	8·50	5	8	7·47

29. *June* 27, 1786.

Barom. 29·94; Therm. 65, at 9 A. M.

Weight	Time	Wt	Time	Wt	Time	Wt	Time
oz dr	sec.	oz	sec.	oz	sec.	oz	sec.
5 12	7·23	9	5·60	20	3·67	64	1·97
6 0	7·05	10	5·32	22	3·48	72	1·88
6 4	6·87	11	5·02	26	3·20	80	1·80
6 8	6·67	12	4·78	30	2·97	90	1·67
7 0	6·47	13	4·58	34	2·77	100	1·60
7 8	6·23	14	4·38	40	2·53	110	1·53
8 0	6·00	16	4·10	48	2·30	120	1·50
8 8	5·90	18	3·87	56	2·10	Thread	broke.

30. *June* 28, 1786.

Barometer 30·06; Thermometer 65.

Experiments with the whole sphere; wt. 8 oz 14 dr.

2	14·5	12	3·35	30	2·07	80	1·22
3	8·55	14	3·1	34	1·95	90	1·15
4	6·75	16	2·85	40	1·8	100	1·07
5	5·75	18	2·7	48	1·65	110	1·02
6	5·1	20	2·6	56	1·5	120	0·97
8	4·3	22	2·45	64	1·4		
10	3·8	26	2·22	72	1·3		

31. The globe required the first weight, or 2 oz, just to put it in motion; and a less weight than 2 oz would not stir it at all. This was owing probably in part to the great weight of the globe, which was made of very thick pasteboard, and partly to the great length of the silk cord, between the barrel and the weight, at first to be moved, which was not less than 23 feet in length. These extra weights causing greater friction and resistance on the machine, it was to be expected that the times of revolution, with the same actuating weights, would be greater, and considerably so when those weights should be very small; but that the difference would gradually diminish as the weights increase; till at length they might become insensible, if the difference

of shape in the afterpart of the figure should not make some sensible difference; a circumstance still remaining to be tried. Accordingly, the experiments show this to be precisely the case, the difference in the times of revolution, between the globe and the hemisphere, being considerable with the small weights, but gradually and continually decreasing, so as nearly to vanish, in comparison with the whole times. So that it appears there is almost no difference, whether a flat side or a round one is hindmost, at least none that is very sensible in the swiftest of these velocities. Those that are set down above, are the mediums of the last 20 revolutions with each weight. Where it is to be noted, that, by mistake in measuring, the centre of the ball was placed $\frac{1}{16}$ of an inch too near the axis; so that the times should be a very little more than are here set down, viz, more in the proportion nearly of $53\frac{1}{4}$ to $53\frac{1}{3}$, which however will not sensibly alter them.

32. *June* 29, 1786.

Weather much the same as yesterday.

Used the cone this day. Its weight was 8 oz 6 dr, and its height $6\frac{5}{8}$ inches, being rather more than its diameter, which was $6\frac{3}{8}$, being the same diameter as the former figures. Tried it with the vortex foremost this day, as follows.

Wt.	Time	Wt.	Time	Wt.	Time	Wt.	Time
oz	sec.	oz	sec.	oz	sec.	oz	sec.
2	13·50	8	4·42	18	2·80	48	1·65
3	8·65	9	4·15	20	2·67	56	1·52
4	6·95	10	3·87	24	2·40	64	1·42
5	5·97	12	3·57	28	2·17	72	1·35
6	5·30	14	3·25	32	2·07	80	1·30
7	4·77	16	3·00	40	1·85	100	1·17

33. It was found that 2 oz was the least weight that would give motion to the machine.—By comparing these numbers with those of the day before, for the globe, it appears that the times to day are nearly equal to, but somewhat

greater than, those of the former day; and consequently, that the resistance on the convex surface of the cone, is nearly equal to, but rather greater than, that on the globe or hemisphere, the diameters being equal, but the height of the cone being a little more than double the height of the hemisphere.

34. The following experiments were next made with the base of the cone foremost.

Wt.	Time	Wt.	Time	Wt.	Time	Wt.	Time
oz	sec.	oz	sec.	oz	sec.	oz	sec.
12	4·95	24	3·40	48	2·37	72	1·93
16	4·25	30	3·05	56	2·20	80	1·85
20	3·77	40	2·60	64	2·02	100	1·65

These times are rather higher than those of the cylinder. But the cone was rather the heavier, and their after parts different.

35. *June* 30, 1786.

Barom. 30·02 ; Therm. 64, at 9 A. M.

Put on the smaller hemisphere, the flat side to go foremost, in order to compare with the large one, to try if the resistance be as the surface, when the axis or radius is the same. The flat surface of this hemisphere is equal to $\frac{1}{8}$ of a square foot, but that of the large one $\frac{2}{9}$, their proportion being as 16 to 9, the radius of the small one's path 52·42, of the large one 53·34; the compound ratio of the surface and path or radius, being 16 × 53·34 to 52·42 × 9, viz, 38 to 21 nearly, or as 11 to 6, but not quite so near.—Note that 6 dr just gave motion to the machine.

1	21·80	7	4·67	20	2·65	56	1·57
2	10·82	8	4·35	24	2·45	64	1·45
3	7·70	10	3·90	28	2·25	72	1·40
4	6·47	12	3·50	32	2·10	80	1·32
5	5·67	14	3·25	40	1·82		
6	5·15	16	3·00	48	1·67		

The following were with the lead only, of the same weight with this small hemisphere, viz, 2 oz 5 dr.

Wt.	Time	Wt.	Time	Wt.	Time	Wt.	Time
oz	sec.	oz	sec.	oz	sec.	oz	sec.
1	8·10	$2\frac{1}{4}$	3·55	4	2·40	8	1·60
$1\frac{1}{4}$	6·00	$2\frac{1}{2}$	3·30	$4\frac{1}{2}$	2·25	9	1·50
$1\frac{1}{2}$	4·95	$2\frac{3}{4}$	3·10	5	2·10	10	1·40
$1\frac{3}{4}$	4·35	3	2·90	6	1·90	11	1·30
2	3·80	$3\frac{1}{2}$	2·60	7	1·70	12	1·25

36. *July* 5, 1786.

Barom. 30·32 ; Therm. 66, at 4 P. M.

Made a thin leaden weight of 8 oz 10 dr, being only 4 dr more than the cone, and 4 dr less than the whole globe, so that it nearly agrees with both. Experiments with it as below.

Wt.		Time	Wt.		Time.	Wt.		Time	Wt.		Time.
oz	dr	sec.	oz	dr	sec.	oz	dr	sec.	oz	dr	sec.
1	8	9·0	2	8	4·2	4	0	2·8	7	0	2·0
1	10	7·0	2	10	4·0	4	4	2·7	8		1·8
1	12	6·4	2	13	3·8	4	8	2·65	9		1·7
1	14	5·9	3	0	3·6	4	12	2·6	10		1·6
2	0	5·3	3	3	3·4	5	0	2·5	11		1·5
2	2	4·9	3	6	3·2	5	8	2·4	12		1·4
2	4	4·6	3	9	3·0	6	0	$2·2\frac{1}{2}$	13		1·35
2	6	4·4	3	12	2·9	6	8	2·1	14		1·3

37. The latter part of the preceding numbers in the times being less than those for the less weight, on May 24, I repeated the experiments with this weight, of 4 oz 3 dr, to get the times for it more exactly. The reason of this difference is, that the arm went only 15 times round in descending on that day, but here 35 times round, the times diminishing a little, all the way to the end, and the times here are the means of the last 10 rounds out of the 35, which are much more exact.

Experiments with the wt. 4 *oz* 3 *dr.*

Weight	Time	Weight	Time	Weight	Time	Weight	Time
oz dr	sec.	oz dr	sec.	oz dr	sec.	oz dr	sec.
2 0	4·6	3 8	3·0	7	1·9	12	1·3
2 4	4·2	4 0	2·7	8	1·75	13	1·25
2 8	3·85	4 8	2·5	9	1·6	14	1·2
2 12	3·5	5	2·3	10	1·5		
3 0	3·3	6	2·1	11	1·4		

38. *July* 6, 1786.

Experiments continued, with the wt. 4 oz 3 dr.

1 4	10·8	1 6	8·4	1 8	7·0	1 12	5·8
1 5	9·8	1 7	7·6	1 10	6·4	1 14	5·3

The following with the Cone, the base foremost, to complete the set.

4 10	9·6	5 8	8·2	9	5·95	100	1·6
4 12	9·3	6 0	7·7	10	5·55	110	1·55
4 14	9·1	7	6·95	12	5·05	120	1·5
5 0	8·9	8	6·3				

When 120 oz was near the bottom, the cord broke.

39. Thus the course of the experiments being now pretty fully extended to various shapes and dimensions of figures, we may next bring into one point of view the weights for all the kinds of figures, and for all the velocities, from 3 to 20 feet per second. And first the irregular numbers just as they were experienced.

Experimented Velocities and Weights.

Veloc. per sec.	Cone, 8 oz 6 dr		Whole globe 8oz 14d	Cylinder 5oz 1dr	Great Hemisphere, 4 oz 3 dr		Small Hemis. 2oz 5dr flat	Lead I. 8 oz 10 dr	Lead II. 4 oz 3 dr	Lead III. 2 oz 5 dr
	Vertex	base			round	flat				
ft.										
3	2·9	4·7	2·8	3·5	2·2	3·8	2·4	1·4	1·3	0·9
4	4·0	6·9	3·9	6·0	3·4	6·1	3·6	1·6	1·5	1·1
5	5·4	9·9	5·2	9·0	4·9	9·2	5·1	1·9	1·8	1·4
6	7·3	13·7	7·1	12·5	6·6	12·8	7·1	2·2	2·0	1·7
7	9·5	18·0	9·2	16·8	8·7	17·0	9·5	2·6	2·4	1·9
8	12·5	22·9	11·3	21·8	11·0	22	12·0	3·1	2·8	2·3
9	15·1	29·0	14·0	27·6	13·3	28	15·1	3·5	3·3	2·7
10	18·0	35·6	16·7	33·5	15·8	34	18·3	4·0	3·8	3·1
11	21·8	41·7	20·7	39·7	19·0	40	22·0	4·4	4·4	3·6
12	25·2	49·9	24·0	47·0	23	48	26·4	5·8	5·1	4·1
13	28·8	58·2	28·0	54	27	56	30·7	6·2	5·7	4·7
14	34·5	65·8	32·0	62	31	64	34·9	7·0	6·5	5·5
15	39·3	78·0	37·2	72	35	73	38·6	7·7	7·2	6·2
16	44·0	90	42·7	84	40	84	43·7	8·5	8·0	6·8
17	48·6	101	48·5	95	45	101	50·4	9·5	8·6	7·6
18	54·2	110	53·3	109	50	112	57·3	10·5	9·4	8·5
19	60·0	126	58·4	125	57	121	62·7	11·3	10·3	9·3
20	66·3		64·0		64	140	72·0	12·0	11·0	10·0

40. But as there are some small irregularities in a few of these numbers, as appears by taking their differences; by correcting these differences so as to make them regular, and then by them correcting the few irregular actuating weights, the more correct table of those numbers will be as follows.

Table of Regular Actuating Weights and Velocities.

Veloc. per sec.	Cone, 8 oz 6 dr		Whole globe 8 oz 14 dr	Cylinder, 5 oz 1 dr	Large Hemisphere, 4 oz 3 dr		Small Hemis. 2oz 5dr	Lead I. 8 oz 10 dr	Lead II. 4 oz 3 dr	Lead III. 2 oz 5 dr
	Vertex	Base			round	flat	flat			
feet										
3	2·9	4·7	2·8	3·5	2·2	3·8	2·4	1·4	1·2	1·0
4	4·1	7·2	4·0	6·0	3·4	6·3	3·6	1·6	1·4	1·2
5	5·5	10·2	5·4	9·0	4·9	9·3	5·1	1·9	1·7	1·5
6	7·2	13·7	7·0	12·5	6·7	12·8	7·0	2·2	2·0	1·8
7	9·2	17·8	9·0	16·6	8·7	16·9	9·2	2·6	2·4	2·1
8	11·6	22·5	11·3	21·3	11·0	21·6	11·7	3·0	2·8	2·5
9	14·3	27·9	14·0	26·7	13·5	27·0	14·6	3·5	3·3	2·9
10	17·3	34·0	17·0	32·8	16·2	33·1	17·8	4·0	3·8	3·4
11	20·7	41·0	20·4	39·6	19·2	40·0	21·4	4·6	4·3	3·9
12	24·4	48·6	24·1	47·2	22·6	47·6	25·4	5·2	4·9	4·4
13	28·4	57·0	28·1	55·6	26·4	56·0	29·7	5·9	5·5	5·0
14	32·8	66·2	32·4	64·8	30·6	65·2	34·4	6·6	6·2	5·6
15	37·5	76·2	37·1	74·8	35·1	75·2	39·4	7·4	6·9	6·2
16	42·6	87·2	42·1	85·8	40·0	86·2	44·8	8·2	7·6	6·9
17	48·0	99·2	47·5	97·8	45·3	98·2	50·5	9·1	8·4	7·6
18	53·8	112·3	53·3	110·9	51·0	111·3	56·6	10·0	9·2	8·4
19	60·0	126·5	59·5	125·1	57·2	125·5	63·1	11·0	10·1	9·2
20	66·6	141·8	66·0	140·4	63·8	140·8	70·1	12·0	11·0	10·0

41. Then, by subtracting the numbers in the last three columns of the leads, from those of their corresponding bodies, the remainders will show the regular set of resistances to the respective figures, as in the following table.—The column of numbers to be subtracted from that of the cylinder, which weighed 5 oz 1 dr, was deduced, by proportion, from the last column but one of the preceding table, which has the lead nearest of the same weight with it.

Table of Regular Mean Resistances to several Figures.

Veloc. per sec.	Small Hemis. flat	Cone.		Whole Globe	Cylinder	Large Hemis.		Ratio of flat to round
		Vertex	Base			convex	flat side	
3	1·4	1·5	3·3	1·4	2·4	1·1	2·6	2·36
4	2·4	2·5	5·6	2·4	4·6	2·1	4·9	2·33
5	3·6	3·6	8·3	3·5	7·3	3·2	7·6	2·38
6	5·2	5·0	11·5	4·8	10·5	4·7	10·8	2·30
7	7·1	6·6	15·2	6·4	14·2	6·3	14·5	2·30
8	9·2	8·6	19·5	8·3	18·4	8·2	18·8	2·30
9	11·7	10·8	24·4	10·5	23·3	10·2	23·7	2·32
10	14·4	13·3	30·0	13·0	28·9	12·4	29·3	2·36
11	17·5	16·1	36·4	15·8	35·2	14·9	35·7	2·40
12	21·0	19·2	43·4	18·9	42·2	17·7	42·7	2·41
13	24·7	22·5	51·1	22·2	50·0	20·9	50·5	2·42
14	28·8	26·2	59·6	25·8	58·5	24·4	59·0	2·42
15	33·2	30·1	68·8	29·7	67·8	28·2	68·3	2·42
16	37·9	34·4	79·0	33·9	78·0	32·4	78·6	2·43
17	42·9	38·9	90·1	38·4	89·2	36·9	89·8	2·43
18	48·2	43·8	102·3	43·3	101·5	41·8	102·1	2·44
19	53·9	49·0	115·5	48·5	114·8	47·1	115·4	2·45
20	60·1	54·6	129·8	54·0	129·2	52·8	129·8	2·46

Here, besides the columns of experimented resistances, in avoirdupois ounces and tenths, for each figure; another column is added at the end, showing the ratios between the correspondent numbers in the two next preceding columns, being the resistances for the round and flat sides of the large hemisphere; which ratios are found by dividing always the greater numbers by the less; showing that they proceed always increasing a very little, as the velocity is increased; the medium among all these being 2·4, instead of 2 only, as expected by the theory of resistances to such figures. But as part of this ratio may be owing to the different shape of the after or hinder sides of the figures, the more correct comparison would be between either the whole sphere and the flat side of the large hemisphere, or the cylinder and the round side of the same large hemisphere, in both of which cases the after parts would be of the same shape, and then the medium ratio would be near 2·3 to 1.—After all, it is to be observed, that the numbers in the preceding table denote, not the true measure of the resistances, but only the weights acting at the circumference of the barrel or axis, while the real resistance of the air acts on the body at the extremity of the arm. To obtain the true resistances then, those numbers must be all divided by the ratios of the corresponding radii or distances at which the weight and the resistance act, viz, by 50·2 for the small hemisphere, and by 51·1 for all the other figures, as determined on the 2nd day of May; this being done, the true measure of the resistances will be as in the table below, for all the velocities from 3 to 20 feet per second of time.

42. *Table of the true Resistances to the Figures in Ounces.*

Veloc. per sec.	Cone. Vertex	Cone. Base	Whole globe	Cylinder	Hemisphere. flat	Hemisphere. round	Small Hemis. flat	Index of the power of the velocity.
feet	oz	oz	oz	oz	oz	oz	oz	
3	·028	·064	·027	·050	·051	·020	·028	
4	·048	·109	·047	·090	·096	·039	·048	
5	·071	·162	·068	·143	·148	·063	·072	
6	·098	·225	·094	·205	·211	·092	·103	
7	·129	·298	·125	·278	·284	·123	·141	
8	·168	·382	·162	·360	·368	·160	·184	
9	·211	·478	·205	·456	·464	·199	·233	
10	·260	·587	·255	·565	·573	·242	·287	
11	·315	·712	·310	·688	·698	·297	·349	2·049
12	·376	·850	·370	·826	·836	·347	·418	2·042
13	·440	1·000	·435	·979	·988	·409	·492	2·036
14	·512	1·166	·505	1·145	1·154	·478	·573	2·031
15	·589	1·346	·581	1·327	1·336	·552	·661	2·031
16	·673	1·546	·663	1·526	1·538	·634	·754	2·033
17	·762	1·763	·752	1·745	1·757	·722	·853	2·038
18	·858	2·002	·848	1·986	1·998	·818	·959	2·044
19	·959	2·260	·949	2·246	2·258	·922	1·073	2·047
20	1·069	2·540	1·057	2·528	2·542	1·033	1·196	2·051
Propor. Numb.	126	291	124	285	288	119	140	2·040

Where the numbers placed at the bottom of all the columns, denote the general mean proportions of their resistances to one another, from the velocity 10, to each of the following ones.

Note. The general mean height of the barometer was 30·1, and of the thermometer 62°; in which circumstances therefore, the ratio of the weight of water to air, was as 840 to 1, or the cubic foot of air weighed nearly $1\frac{1}{5}$ oz avoirdupois.

It may be repeated also, that the diameter of the small hemisphere was $4\frac{3}{4}$ inches, and consequently the area of its base, or great circle, was $17\frac{3}{4}$ square inches, or $\frac{1}{8}$ of a square foot nearly. But the common diameter of all the other figures was $6\frac{3}{8}$ inches, and therefore the area of the circular base, was just 32 square inches, or $\frac{2}{9}$ of a square foot. Also, the altitude of the cone was $6\frac{5}{8}$ inches; its side is therefore inclined to the axis in an angle of 25° 42′.

43. From the preceding table of resistances, several practical inferences may be drawn; such as the following.

(1). That the resistance is nearly as the surface; the resistance increasing but a very little above that proportion in the greater surfaces. Thus, by comparing together the numbers in the 6th and 8th columns, for the bases of the two hemispheres, the areas of which are in the proportion of $17\frac{3}{4}$ to 32, or as 5 to 9 very nearly; it appears that the numbers in those two columns, expressing the resistances, are nearly as 1 to 2, or as 5 to 10, as far as to the velocity of 12 feet; after which the resistances on the greater surface increase gradually more and more above that proportion; and the mean resistances are as 140 to 288, or as 5 to $10\frac{2}{7}$, so far as these experiments have been extended. Nearly the same proportions result from the comparison of the numbers in other columns with similar surfaces. Now, the ratio of the surfaces being that of 5 to 9; and the ratio of the resistances as 5 to $10\frac{2}{7}$; it appears that the latter exceeds the former in the ratio of 9 to $10\frac{2}{7}$, or of 1 to $1\frac{1}{7}$; or of 7 to 8. Hence then, to find the resistance to any surface, from having given that of a similar smaller one, with an equal velocity; first raise the given resistance in the proportion of the less surface to the greater; then increase the result by $\frac{1}{7}$ part, or in the ratio of 7 to 8, which will give the resistance for the larger surface, nearly. And, reverse wise, to find the resistance on a smaller body, from that of a like larger one, with equal velocity.—We shall afterwards enquire whether this proportion will answer in very high velocities also.

(2). The resistance to the same surface, with different velocities, is, in these slow motions, nearly as the square of the velocity; but gradually increasing more and more above that proportion, as the velocity increases. This will appear, on trial, from all the columns; and the index of the power of the velocity is set down in the last column, for the resistances in the 4th column, for the whole globe, by comparing that for the velocity 10, with each of the following ones, 11, 12, 13, &c; where, after the first three or four, the numbers gradually and slowly increase, and would doubtless continue to do so to a very great extent. The medium among those here set down, is 2·040; whence it appears, that in these slow motions, the resistance to the same body, is nearly as the 2·04 power of the velocity, or indeed as the square of it nearly.

Veloc	Resist.	Index
10	255	
11	310	2·049
12	370	2·042
13	435	2·036
14	505	2·031
15	581	2·031
16	663	2·033
17	752	2·038
18	848	2·044
19	949	2·047
20	1057	2·051

An easy method of determining those indices, is as follows: Make a small tablet, as in the margin, placing, in the first column, the velocities from 10; in the 2nd column the correspondent resistances from any column of the former table, as suppose those for the whole globe. Then, to fill up the 3rd column, put $r = 255$ the first resistance, $r' =$ any other, as the 2nd 310; and $x =$ the required index of the power of the velocities, which shall be proportional to the resistances; that is, so as that $10^x : 11^x :: r : r'$. Dividing the first two terms by 10, and the latter two by r, the proportion becomes $1 : 1{\cdot}1^x :: 1 : \frac{r'}{r}$, consequently $1{\cdot}1^x = \frac{r'}{r}$; the logarithm of this is $x \times \log.\ 1{\cdot}1 = \log.\ r' - \log.\ r$; and hence $x = \frac{\log.\ r' - \log.\ r}{\log.\ 1{\cdot}1} = 2{\cdot}049$, by using the aforesaid numbers for r and r'. In like manner, taking, in the denominator of the general expression for x, the successive numbers 1·2, 1·3, 1·4, &c, to 2·0; and in the numerator the successive values of r', viz, 370, 435, &c, that of r being constantly $= 255$; all the numbers in the 3rd column will be easily found.

(3.) The round ends, and sharp ends, of solids, suffer less resistance than the flat or plane ends, of the same diameter: thus, the cylinder, and the flat ends of the hemisphere and cone, have more resistance than the round or sharp ends of the same. But the sharper end has not always the smaller resistance: thus, the round side of the hemisphere has less resistance than the pointed end of the cone.

(4.) When the hinder parts of bodies are of different forms, the resistances are different, though the fore parts should be exactly alike and equal; owing probably to the different pressures of the air, on the after parts. Thus, the resistance to the fore part of the cylinder, is less than that on the equal flat surface of the cone, or of the hemisphere; because the hind part of the cylinder is more pressed or pushed forward, by the following air, than those of the other two figures: also, for the same reason, the base of the hemisphere shows a less resistance than that of the cone, and the round side of the hemisphere less than the whole sphere.

(5.) The resistance on the base of the hemisphere, is to that on the convex side, or on the whole sphere, nearly as $2\frac{2}{5}$ to 1, instead of 2 to 1, as assigned by theory. And the experimented resistance, in each of these cases, is nearly $\frac{1}{4}$ part more than that which is determined by the theory.

(6.) The resistance on the base of the cone, is to that on the vertex, nearly as 2·3 to 1. And almost in the same ratio is radius to the sine of the angle of inclination of the side of the cone to its path or axis. So that, in this instance, the resistance is directly as the sine of the angle of incidence, the transverse section being the same.

(7). Hence may be determined what will be the altitude of a column of air, whose pressure shall be equal to the resistance of a body, moving through it with any velocity. Thus,

Let $a =$ the area of the section of the body, similar to any of those in the table, perpendicular to the direction of motion;

$r =$ the resistance to the velocity in the table; and

$x =$ the altitude sought, of a column of air, whose base is a, and its pressure r.

Then $ax =$ the content of the column in feet,
and $1\frac{1}{5}ax$ or $\frac{6}{5}ax$ its weight in ounces;
therefore $\frac{6}{5}ax = r$, and $x = \frac{5}{6} \times \frac{r}{a}$ is the altitude sought in feet, namely, $\frac{5}{6}$ of the quotient of the resistance of any body divided by its transverse section; which is a constant quantity for all similar bodies, however different in magnitude, since the resistance r is nearly as the section a, as was found in art. 1. When $a = \frac{2}{9}$ of a foot, as in all the figures in the foregoing table, except the small hemisphere: then, $x = \frac{5}{6} \times \frac{r}{a}$, becomes $x = \frac{15}{4}r$, where r is the resistance in the table, to the similar body.

If, for example, we take the convex side of the large hemisphere, whose resistance is ·634 oz to a velocity of 16 feet per second, then $r = {\cdot}634$, and $x = \frac{15}{4}r = 2{\cdot}3775$ feet, is the altitude of the column of air whose pressure is equal to the resistance on a spherical surface, with a velocity of 16 feet. And to compare the above altitude with that which is due to the given velocity, it will be $32^2 : 16^2 : : 16 : 4$, the altitude due to the velocity 16; which is near double the altitude that is equal to the pressure. And as the altitude is proportional to the square of the velocity, therefore, in small velocities, the resistance to any spherical surface, is equal to the pressure of a column of air on its great circle, whose altitude is $\frac{19}{32}$ or ·594 of the altitude due to its velocity.

But if the cylinder be taken, whose resistance $r = 1\cdot526$: then $x = \frac{15}{4}r = 5\cdot72$; which exceeds the height, 4, due to the velocity, in the ratio of 23 to 16 nearly. And the difference would be still greater, if the body were larger, and also if the velocity were more.

(8). Also, if it be required to find with what velocity any flat surface must be moved, so as to suffer a resistance just equal to the whole pressure of the atmosphere:

The resistance on the whole circle whose area being $\frac{2}{9}$ of a foot, is $\cdot051$ oz, with the velocity of 3 feet per second; it is $\frac{1}{9}$ of $\cdot051$, or $\cdot0056$ oz only, with a velocity of 1 foot. But $2\frac{1}{2} \times 13600 \times \frac{2}{9} = 7555\frac{5}{9}$ oz, is the whole pressure of the atmosphere. Therefore, as $\sqrt{\cdot0056} : \sqrt{7556} :: 1 : 1162$ nearly, which is the velocity sought. Being almost equal to the velocity with which air rushes into a vacuum.

(9). Hence may be inferred the great resistance suffered by military projectiles. For, in the table, it appears, that a globe of $6\frac{3}{8}$ inches diameter, which is equal to the size of an iron ball weighing 36lb, moving with a velocity of only 16 feet per second, meets with a resistance equal to the pressure of $\frac{2}{3}$ of an ounce weight; and therefore, computing only according to the square of the velocity, the least resistance that such a ball would meet with, when moving with a velocity of 1600 feet, would be equal to the pressure of 417lb, and that independent of the pressure of the atmosphere itself on the fore part of the ball, which would be 487lb more, as there would be no pressure from the atmosphere on the hinder part, in the case of so great a velocity as 1600 feet per second. So that the whole resistance would be more than 900lb to such a velocity!

(10). Having said, in the last article, that the pressure of the atmosphere is taken entirely off the hinder part of the ball moving with a velocity of 1600 feet per second; which must happen, when the ball moves faster than the particles of air can follow, by rushing into the place quitted and left void by the ball, or when the ball moves faster than the air rushes into a vacuum from the pressure of the incumbent air: let us therefore inquire what this velocity is. Now the velocity with which any fluid issues, depends on its altitude above the orifice, and is indeed equal to the velocity acquired by a heavy body in falling freely through that altitude. But, supposing the height of the barometer to be 30 inches, or $2\frac{1}{2}$ feet, the height of a uniform atmosphere, all of the same density as at the earth's surface, would be $2\frac{1}{2} \times 13{\cdot}6 \times 833\frac{1}{3}$ or 28333 feet; therefore $\sqrt{16} : \sqrt{28333} :: 32 : 8\sqrt{28333} = 1346$ feet, which is the velocity sought. And therefore, with a velocity of 1600 feet per second, or any velocity above 1346 feet, the ball must continually leave a vacuum behind it, and so must sustain the whole pressure of the atmosphere on its fore part, as well as the resistance arising from the *vis inertia* of the particles of air struck by the ball.

(11). On a review of the whole of the premises, we find that the resistance of the air, as determined by the foregoing experiments, differs very widely, both in respect to its quantity on all figures, and in regard to the proportions of its action on oblique surfaces, from the same actions and resistances as assigned by the most plausible and imposing theories, which have heretofore been delivered, and confided in by philosophers. Hence it may be concluded, that all the speculative theories on the resistance of the air, hitherto laid down, are very erroneous; and that it is from experiments, carefully and skilfully executed, that a rational hope can be grounded, of deducing and establishing a true and useful theory of the actions of forces, so intimately connected with the numerous and important concerns in human life.

Proceed we now to relate the further progress of our experiments with the whirling-machine, on other bodies, and in other positions.

EXPERIMENTS OF THE YEARS 1787 and 1788.

July 31, 1787.

44. By comparing the experimented resistance on the vertex of the cone, with the theory, it appears that it is nearly equal to the resistance of the sphere; whereas, by the received theory, it ought to be only about $\frac{2}{5}$ of that of the sphere. I therefore this day tried these two bodies again, and found the results nearly the same as before. With the 32 oz weight, the mediums among the times of a revolution, were thus: viz,

This year they were,	Last year they were
for the cone 2·10″	2.07″
for the globe 2·02	2·01, nearly the same.

So that it seems difficult to account for so great a difference between theory and experiment, in the case of the cone's vertex.

Aug. 5, 1787.

Fine, clear, warm Weather.

45. Performed experiments with the following bodies, and found the medium of the times of revolution as follows.

Large Globe	The Cone. vertex	The Cone. base	Samll Hemis. base	Samll Hemis. conv.	Lead of 2 oz 5 dr
2″	2·05″	2·9″	2·3	1·6	1·6

The actuating weight was 32 oz for each figure, and 8 oz for the thin lead weight, the same as June 30, 1786.

EXPERIMENTS IN 1788 WITH THE WHIRLING-MACHINE.

July 23, 1788.

46. Prepared the machine to make experiments with figures of shapes different from the foregoing ones. Providing a thin rectangular plate of brass, to fix on the arm of the machine; its weight 11 oz $0\frac{3}{4}$ dr, or $11\frac{1}{21}$ oz, and its dimensions 8 inches by 4, consequently its area was 32 square inches, the same as the plane surface of the cone and large hemisphere, before employed. It was adapted for fitting on the end of the arm in both directions, viz, in the direction both of the length and of the breadth, to try the effect of the same surface in both positions. It was also contrived to incline the surface in any degree to the direction of motion, to try the resistance at all angles of inclination. When fitted on with its length in the direction of the arm, the distance of its centre, from the axis of motion, was $53\frac{3}{4}$ inches; and the same distance also when fitted on the other way.

July 24, 1788.

Barometer 30·0; Thermometer $70\frac{1}{2}$.

47. Tried the experiments this day with the plane fitted on with its length in the direction of the arm, its centre being distant $53\frac{3}{4}$ inches from the axis of motion. The plane was inclined in various angles, as expressed below; and the actuating weight was 32 oz or 2lb each time.

Table of the Times of Revolution, the Plane lengthwise.

0° or horizontal	5°	10°	20°	30°	40°	45°	50°	60°	70°	80°	90° or vertical
1·00″	1·17″	1·34″	1·67″	1·98″	2·27″	2·40″	2·54″	2·78″	2·94″	3·00″	3·02″

In the first line of this table, are the angles in which the plane was inclined to the horizon, or to its path, sometimes at 5 degrees, and sometimes at 10°, difference from each other; the first position being 0°, or horizontal, with its edge to the air; and the last 90°, or the plane vertical, with its face directly opposed to the path or resistance of the air. In the second line are the intervals of time in performing each revolution, expressed in seconds and hundredth parts; each number, as usual, being a medium of several repetitions.

Note, The area of the plane is 32 square inches, or $\frac{2}{9}$ sq. foot, and = the circle of the large hemisphere.

The radius of revolution to centre of the plane $53\frac{3}{4}$ inches.

Circumference described by its centre in 1 revolution 337·7 inc. or 28·14 feet.

Ratio of radius of axis to radius of revolution, as 1 to 51·51, accounting the radius of axis to the middle of the thread, 1·044 inches.

July 25, 1788.

Barometer 60·10; Thermometer 66.

48. Used the same brass plane again, set at various angles as before, but fixed on the contrary way, viz, the breadth or shorter dimension in the direction of the arm; but still having its centre at the same distance of $53\frac{3}{4}$ inches from the centre of motion; to try if there be any difference by such change of position; the actuating weight being still the same 32 ounces.

Times of Revolution, with the Plane crosswise.

0° or horiz.	5°	10°	20°	30°	40°	45°	50°	60°	70°	80°	90° or vert.
1·00″	1·17″	1·35″	1·69″	2·00″	2·30″		2·57″	2·80″	2·93″	3·00″	3·03″

Here, as before, the first line contains the inclination of the plane to the direction of motion, and the second the medium times of revolution. The inclination of 45° was not used this day; but it was tried in two different ways the day before, viz, inclined 45° backwards, thus ∖, and then inclined 45° forward, thus ∕, when the result was the same in both cases: whence it may be concluded, that the resistance is the same, whether the plane is inclined forward or backward, at each angle of inclination, as indeed it might be expected.

By comparing the experiments of this day, with those of the day before, it appears that the differences are very small indeed; but those of this day are rather longer intervals, if any thing; but the difference is so small, and doubtful, that the times may be accounted equal.

49. After the above experiments, the following course was then instituted with a thin bit of lead, on the end of the arm, equal to the weight of the brass plane, and which, its resistance being very small, will give nearly the weights to be deducted from all the above, on account of the friction of the machine and the resistance on the arm, viz, with the

actuating wts. in oz.	8	10	12	16	20	24	28	32
time of each revolut.	3·33″	2·50	2·00	1·54	1·35	1·20	1·09	1·00

And from these, by interpolation are deduced the following.

Incl.	0	5°	10°	20°	30°	40°	50°	60°	70°	80°	90°
Wts.	oz. 32	24·8	19·5	15·5	12·7	10·9	9·9	9·3	8·9	8·7	8·6
Times	1·00′	1·17″	1·35″	1·68″	1·99″	2·28″	2·56″	2·79′	2·94′	3·00″	3·02″

50. And hence the following general table.

Inclin. of the plane.	Time of one revolution.	Actuating wt. with plane	Actuating wt. without plane	Diff. or resistance.	Ditto reduced to arm.	Velocity per second.
		oz	oz	oz	oz	feet
0°	1·00″	32	32·0	0·0	·000	28·1
5	1·17	32	24·8	7·2	·140	24·1
10	1·35	32	19·5	12·5	·243	20·8
20	1·68	32	15·5	16·5	·320	16·8
30	1·99	32	12·7	19·3	·371	14·1
40	2·28	32	10 9	21·1	·410	12 4
45	2·42	32	10·4	21·6	·420	11·6
50	2·56	32	9·9	22·1	·429	11·0
60	2·79	32	9·3	22·7	·441	10·1
70	2·94	32	8·9	23·1	·448	9·6
80	3·00	32	8·7	23·3	·452	9·4
90	3·02	32	8·6	23·4	·454	9·3
1	2	3	4	5	6	7

Here, the numbers in the 6th column, in thousandth parts of an ounce, are equal to the resistance of the plane, placed at the angle on the same line in the first column, when making one revolution, or describing the space $28\frac{1}{7}$ feet, in the time or number of seconds in the 2nd column, or when moving at the rate denoted by the numbers in the 7th column.

From what has been done, in the last two days, it appears, that there is no sensible difference between the two different ways of putting on the plane, viz, the length or breadth in direction of the length of the arm; provided its centre be but at the same distance from the axis, in both cases.

But now to obtain the law of resistance, for the several angles of inclination, with one and the same velocity, it was proper to institute a fresh set of experiments, as they here follow.

July 31, 1788.

Barometer 29·91; Thermometer 72.

51. The object of this day's experiments was, to find the resistance on the same plane, at every angle of elevation, when the velocity, as for example, is 20 feet per second. I therefore began as follows, putting on different weights, to bring the velocity to about 20 feet in a second.

Angle with horiz.	Actuating wts.	Time of one revolution.	Veloc. per second.
	oz	sec.	feet
0°	19	1·50	18·76
·	20	1·43	19·68
5	22	1·43	19·68
·	23	1·33	21·16
10	24	1·43	19·68
·	25	1·40	20·10
20	26	1·76	16·00
·	27	1·73	16·27
·	28	1·67	16·85
·	29	1·65	17·06
·	30	1·58	17·81
·	32	1·56	18·04
·	34	1·52	18·51
·	36	1·48	19·02
·	38	1·44	19·54
·	40	1·36	20·69
30	44	1·60	17·59
·	48	1·54	18·27
·	52	1·43	19·68
·	56	1·40	20·10
40	64	thread	broke

The thread having broken with the last weight, of 64 oz, the experiments could not be further prosecuted for the proposed velocity, 20 feet. I was therefore obliged to be content with the case of a smaller velocity, as that of 12 feet for example, which would not require so large an actuating weight; the results of which are as follow.

Angle with horiz.	Actu-ating wts.	Time of 1 revol.	Veloc. per sec.	Diffs. of wts.	Resist. to 12 ft. veloc.	Ditto reduced.	Ratio to the last.	Sines of the angles
	oz	sec	feet	oz	oz	oz		
0°	10	2·50	11·25	0	0	0	0	0
5	$10\frac{1}{4}$	2·67	10·54	$0\frac{1}{4}$	0·81	·016	19	87
·	$10\frac{3}{4}$	2·43	11·58	$0\frac{3}{4}$				
10	12	2·50	11·25	2	2·28	·044	52	173
20	15	2·73	10·31	5	6·84	·133	158	342
·	16	2·50	11·25	6				
30	24	2·37	11·87	14				
·	26	2·26	12·45	16	14·31	·278	331	500
·	28	2·19	12·85	18				
40	32	2·40	11·72	22				
·	34	2·30	12·23	24	23·09	·448	533	643
·	36	2·26	12·45	26				
50	39	2·40	11·72	29				
·	40	2·39	11·77	30	31·34	·609	724	766
·	41	2·38	11·83	31				
60	43	2·50	11·25	33	37·60	·730	868	866
70	44	2·58	10·91	34	41·49	·805	957	940
·	45	2·55	11·04	35				
80	46	2·60	10·88	36	43·06	·836	994	985
·	47	2·52	11·14	37				
90	48	2·50	11·25	38	43·35	·841	1000	1000
1	2	3	4	5	6	7	8	9

52. Having, by the mediums of several trials, found the times corresponding to each revolution, in the 3rd column of this table, corresponding to the weights in the 2nd column, and angles of inclination in the first column; the circumference of revolution, viz, $28\frac{1}{7}$ feet, being divided by these times, gives the velocity per second, as placed in the 4th column.

Then the first weight, of 10 oz, when the very thin edge of the plane only was opposed to the air, being deducted from every weight in the 2nd column, leaves the weights in the 5th column, which are to be considered as the actuating weights at the barrel of the axis, very nearly.

In the 6th column are the several resistances to a velocity of 12 feet per second, as deduced from those in the 4th column, to which number these are nearly equal, by stating the resistances to be proportional to the 2·04 power of the velocity, agreeably to the rate we have before determined for slow motions, that are so nearly equal to each other.

In the 7th column, the resistances of the 6th column are reduced, in the proportion of the distance of the plane to the distance of the actuating weight; and these are the true measures of the resistances, in thousandth parts of an ounce, to the velocity 12 feet in a second of time, for each position of the plane.

In the 8th column are set the ratios of all these resistances, to the last or greatest of them; which are obtained by dividing each of them by that greatest; being done in order to compare them with the sines of the same angles of inclination, as placed in the 9th or last column Whence it appears, that the resistance is in no wise proportioned to those sines: for, from the direct vertical position, or 90 degrees, the sine of the angle of inclination decreases faster than the resistance, till at the angle of 60° inclination, where they are again equal. After which, the resistance decreases faster than the sines; but no where however so fast as the squares of the sines, and much less of their cubes. So that the resistance, it appears, is in practice not as any single power of the sines, neither 1st, nor 2nd, nor 3rd power of them.

53. But, if we would search out some powers or function of the sines, that should be proportional to the resistances, it is manifest that the exponent must be some variable quantity, which at 60° must be $= 1$, or nearly so; below 60°, it must be greater than 1, but less than 2, and above 60° it must be between 1 and 0. Now the cosines of the angles increase from 90° down to 0, in such sort, that their doubles have the above requisite property, viz, that at 60°, $2c$ is $=1$; above 60°, $2c$ is between 1 and 0; and below 60°, $2c$ is between 1 and 2; where c denotes the cosine of the angle of inclination. Let us compute then the proportions of s^{2c} for

the several angles of inclination, where s denotes the sine, in order to compare with the resistances, and we shall find the values of s^{2c}, for the several angles, to be thus:

Angles	Values of s^{2c}	Exper. Resist.	Diffs.
5°	8	19	11
10	32	52	20
20	133	158	25
30	302	331	29
40	508	533	25
50	710	724	14
60	866	868	2
70	958	957	-1
80	995	994	-1
90	1000	1000	0

Here it appears, that the values of s^{2c} agree with the ratios of the resistances, from 90° to 60° very nearly; but that from 60° to 30°, the differences increase, and from 30° to 0, they decrease again, the greatest difference being at 30°, where it is $\frac{1}{11}$ of the whole.

54. Instead of the index $2c$, in the function then, it is likely that we shall succeed better if we assume the indefinite one nc, by adapting its result to the resistance for the experimented angle of 30°, that is, to ascertain what n must be, so as that the function s^{nc} may agree with the resistance at 30°, viz, by making $s^{nc} = 331$. Now this equation in logarithms is $nc \times \log. s = \log. 331$, and hence $n = \frac{\log. 331}{c \times \log. s} = 1{\cdot}842$ nearly. Then taking always $s^{1{\cdot}842c}$, there results the following numbers:

Angles	Values of $s^{1{\cdot}842c}$	Exper. resist.	Diffs.
5°	11	18	7
10	42	52	10
20	156	158	2
30	331	331	0
40	536	533	-3
50	730	724	-6
60	876	868	-8
70	962	957	-5
80	995	994	-1
90	1000	1000	0

Here the differences being all very small, it may be concluded that the function $s^{1{\cdot}842c}$ is an expression for the resistance, sufficiently near the truth, for all the angles of inclination. And this being multiplied by ·841, to bring it to ounces, will give $\cdot 841 s^{1{\cdot}842c}$ for the resistance, in ounces, for any angle, whose sine is s, and cosine c, for the velocity of 12 feet in a second, on a plane surface of 32 square inches, or $\frac{2}{9}$ of a square foot.

Aug. 11, 1788.

Barometer 30·08 ; Thermometer 60.

55. The occasion of this day's experiments was as follows: On comparing the last experiments, made with the parallelogram inclined in various angles, with those formerly made with the cone, the vertex foremost, the surface of which I thought might be considered as expanded or unrolled in a triangle of an equal area, and inclined in the same angle. But the resistances not turning out to be in the proportion of the two surfaces, I imagined that the difference might result from the diversity in the shape of the two figures, or surfaces compared, the one being a plane rectangle, and the other a conical surface, or as it were a triangle rolled up.

56. I therefore procured an equilateral triangular brass plane, nearly of the same area and weight as the parallelogram last used, viz, the triangle being $31\frac{2}{7}$ square inches, or nearly $\frac{2}{9}$ of a square foot, and performed the following experiments with it, inclining the plane to the direction of motion in all angles, differing by 5° at each time, and at each inclination turning the vertex of the triangle both forewards and backwards, so that it might go both foremost and hindmost; but, as there was no difference in the resistance between the two positions, the numbers are set down only for one of them, the same numbers serving for both. The middle line of the plane was placed at $53\frac{3}{4}$ inches from the centre of motion; and therefore the velocity of that line, is to the velocity of the actuating weight, as 51·51 to 1. The weights, for each inclination, were gradually changed, so as to bring the plane to make 2 uniform revolutions in every 5 seconds of time, in which case the velocity of the plane was $11\frac{1}{4}$ feet per second, or rather 11·26 feet.

57. Having therefore placed the several angles in the first column of the following table, and the actuating weights in the 2nd column, which gave 2 revolutions in 5 seconds, in the 3rd column are set the differences between the first weight, when horizontal, and each of the other following

weights, which remainders denote the resistances without the friction and resistance on the arm, for a velocity of 11·26 feet. Then these are raised in the ratio of $11{\cdot}26^{2{\cdot}04}$ to $12^{2{\cdot}04}$, or 7 to 8 nearly, for the resistances to 12 feet velocity, which are placed in the 4th column. These last numbers are again increased in the ratio of the surface $31\frac{2}{7}$ of this triangle, to 32 the surface of the former plane, that is, in the ratio of 43 to 44, or 1 to $1\frac{1}{43}$, which increased numbers are set in the next or 5th column. Then—

Angles with direc.	Actuating wts.	Diffs. of ditto	Ditto for 12 feet velocity	Ditto for 32 inches area	Resist. reduced	Ratios to the last	Sines of the angles
	oz	oz	oz	oz	oz		
0°	10	0·0	0·0	0·0	·000	·000	·000
5	10·7	0·7	0·8	0·8	·015	·018	·087
10	11·8	1·8	2·0	2·0	·039	·046	·174
15	13·4	3·4	3·9	4·0	·077	·091	·259
20	15·8	5·8	6·6	6·8	·132	·156	·342
25	19·0	9·0	10·3	10·5	·204	·241	·423
30	23·0	13·0	14·8	15·1	·294	·347	·500
35	27·3	17·3	19·7	20·1	·390	·461	·574
40	31·6	21·6	24·7	25·2	·490	·579	·643
45	35·4	25·4	29·0	29·7	·577	·682	·707
50	38·6	28·6	32·7	33·5	·650	·768	·766
55	41·1	31·1	35·5	36·3	·705	·833	·819
60	43·0	33·0	37·7	38·6	·750	·886	·866
65	44·5	34·5	39·4	40·3	·783	·925	·906
70	45·7	35·7	40·8	41·8	·809	·956	·940
75	46·4	36·4	41·6	42·6	·827	·977	·966
80	46·9	36·9	42·1	43·1	·837	·989	·985
85	47·2	37·2	42·5	43·5	·845	·998	·996
90	47·3	37·3	42·6	43·6	·846	1·000	1·000
1	2	3	4	5	6	7	8

these are divided by 51·51, to reduce the weights from the barrel to the extremity of the arm, and the quotients are the true measures of the resistances, which are placed in the next column 6.—Lastly, each of these being divided by the greatest, 846, give their proportions in numbers easily to be compared with the sines of the angles: and these proportionals are placed in the 7th column; having the correspondent sines of the angles of inclination set in the 8th or last column.

On a slight comparison it soon appears, that the numbers for the triangular shape, are nearly the same as those for the parallelogram, of equal area; evincing that the difference in the shape of the plane makes no material difference in the resistance, the area being equal.

58. By interpolation from the foregoing numbers, the following table is deduced, for every single degree of inclination, accompanied with the correspondent sines of the same angles.

Angle	Ratios	Sines	Angl.	Ratios	Sines	Angl.	Ratios	Sines
1°	3	17	31°	369	515	61°	895	875
2	6	35	32	392	530	62	903	883
3	10	52	33	415	545	63	911	891
4	14	70	34	438	559	64	918	899
5	18	87	35	461	574	65	925	906
6	23	105	36	484	588	66	932	914
7	28	122	37	507	602	67	938	921
8	34	139	38	531	616	68	944	927
9	40	156	39	555	629	69	950	934
10	46	174	40	579	643	70	956	940
11	53	191	41	602	656	71	961	946
12	61	208	42	623	669	72	966	951
13	70	225	43	643	682	73	970	956
14	80	242	44	663	695	74	974	961
15	91	259	45	682	707	75	977	966
16	102	276	46	700	719	76	980	970
17	114	292	47	718	731	77	983	974
18	127	309	48	735	743	78	985	978
19	141	326	49	752	755	79	987	982
20	156	343	50	768	766	80	989	985
21	171	358	51	783	777	81	991	988
22	187	375	52	797	788	82	993	990
23	204	391	53	810	799	83	995	993
24	222	407	54	822	809	84	997	995
25	241	423	55	833	819	85	998	996
26	261	438	56	845	829	86	999	998
27	282	454	57	856	839	87	999	999
28	303	469	58	866	848	88	999	999
29	325	481	59	876	857	89	1000	1000
30	347	500	60	886	866	90	1000	1000

Where the numbers in the 2nd column, when multiplied by 84, and divided by 1000, will give the resistance in decimals of an ounce, to a plane of 32 square inches area, moved with the velocity of 12 feet in a second of time, meeting the air in the several angles shown in the first column of the table: and the general formula for every case of that plane, as before, is $84s^{nc}$, where s denotes the sine, and c the cosine of the angle of inclination, also $n = 1{\cdot}842$, or $1{\cdot}84$ only. For other surfaces and velocities, the formula will vary nearly as the surfaces, or but little higher, about $\frac{1}{7}$, and nearly as the 2·04 power of the velocity. Or, for any other greater plane a, and velocity v, the formula will be nearly $\cdot 03av^{2\cdot 04}s^{1\cdot 84c}$ feet.—For water, or any other fluid, different from air, this theorem will be varied in proportion to the density of the fluid.

59. Though the figure of the plane makes no sensible difference in the resistance; yet the action on a plane will not apply to such curve surfaces as those of a cone or a sphere, as clearly appears by the table of experiments in page 189; where the convexity of the hemisphere, on a surface equal to double its base, has less than half the resistance; and the cone, with a surface of nearly 74 square inches, at an angle of 25°42′, with the path, suffers a resistance far less in proportion to the plane at an equal angle; for, the ratio of the surfaces is that of 74 to 32, much more than double, while that of the resistances, 376 to 255, is less than 3 to 2.